EQUATIONS FOR CALCULATING THE LEAKAGE THROUGH COMPOSITE LINERS

The attached paper presents empirical equations for calculating the expected rate of liquid flow through composite liners due to geomembrane defects. Equations are provided that are applicable to composite liners containing either compacted clay liners (CCLs) or geosynthetic clay liners (GCLs). Note in the equations that a key parameter to leakage through geomembrane defects in composite liners is the hydraulic conductivity of the underlying clay liner. Thus, if the hydraulic conductivity of a GCL is less than that of a CCL, then the leakage through the composite liner will be less using a GCL instead of a CCL.

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Technical Paper by J.P. Giroud

EQUATIONS FOR CALCULATING THE RATE OF LIQUID MIGRATION THROUGH COMPOSITE LINERS DUE TO GEOMEMBRANE DEFECTS

ABSTRACT: Equations available to date for calculating the rate of liquid migration through a composite liner due to geomembrane defects require the use of graphs to obtain the value of one of the terms of the equations for the case where the liquid head is larger than the thickness of the low-permeability soil component of the composite liner. In this paper, it is shown that the terms that require graphs can be expressed analytically, which leads to a new set of equations that provides an entirely analytical means of calculating the rate of liquid migration through composite liners. This new set of equations is particularly useful when the liquid head is large compared to the thickness of the low-permeability soil component of the composite liner, which is often the case when the low-permeability soil associated with the geomembrane to form a composite liner is a geosynthetic clay liner. A numerical example is given.

KEYWORDS: Liquid migration, Leachate migration, Leakage, Composite liner, Equations.

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1 INTRODUCTION

1.1 Purpose of this Paper

Equations are available to calculate the rate of liquid migration through a composite liner, due to geomembrane defects, when the liquid head on top of the liner is small compared to the thickness of the low-permeability soil component of the composite liner, whether the defect is small (Giroud et al. 1989) or large (Giroud et al. 1992). Equations are also available for the case where the head of liquid on top of the liner is large compared to the thickness of the low-permeability soil component of the composite liner (Giroud et al. 1992,1994); however, in such a case, graphs are necessary to obtain the value of one of the terms of the equations, which is cumbersome.

In this paper, it is shown that the graphs can be replaced by equations, which leads to an entirely analytical method for the evaluation of the rate of leachate migration through a composite liner, regardless of the head of liquid on top of the liner.

1.2 Composite Liner

A composite liner is a liner that consists of two or more components. In the context of this paper, the term composite liner will be used for liners that consist of two components, a geomembrane and a low-permeability soil, the geomembrane being on top of the low-permeability soil.

The low-permeability soil component of a composite liner is generally either a compacted clay liner (CCL) or a geosynthetic clay liner (GCL). The thickness of a CCL is typically between 0.3 and 1.5 m whereas the thickness of a hydrated GCL depends on the compressive stress applied during hydration and is typically between 5 and 10 mm, i.e. on the order of 100 times less than the thickness of a CCL. The hydraulic conductivity of both CCLs and GCLs depends on the nature of the material, the nature of the liquid, and the applied compressive stress; when the liquid is water or a leachate that does not affect the hydraulic conductivity of clay, including bentonite, the hydraulic conductivity of a CCL is typically between 1×10^{-10} and 1×10^{-9} m/s whereas the hydraulic conductivity of a GCL is typically between 5×10^{-12} and 5×10^{-11} m/s, i.e. on the order of 10 to 100 times less than the hydraulic conductivity of a CCL.

1.3 Liquid Migration Through a Composite Liner

Since an intact geomembrane has an extremely low permeability, most of the liquid migration through a composite liner occurs through geomembrane defects. In this paper, the only mechanism of liquid migration that is considered is flow through geomembrane defects. The liquid considered herein is water or any aqueous solution such as leachate from municipal or hazardous solid waste landfills.

If there is a defect in the geomembrane component of a composite liner, the liquid passes first through the geomembrane defect, then it flows laterally some distance between the geomembrane and the low-permeability soil, and, finally it infiltrates into and through the low-permeability soil layer which is the second component of the composite liner (Figure 1). Flow in the space between the geomembrane and the low-perme-

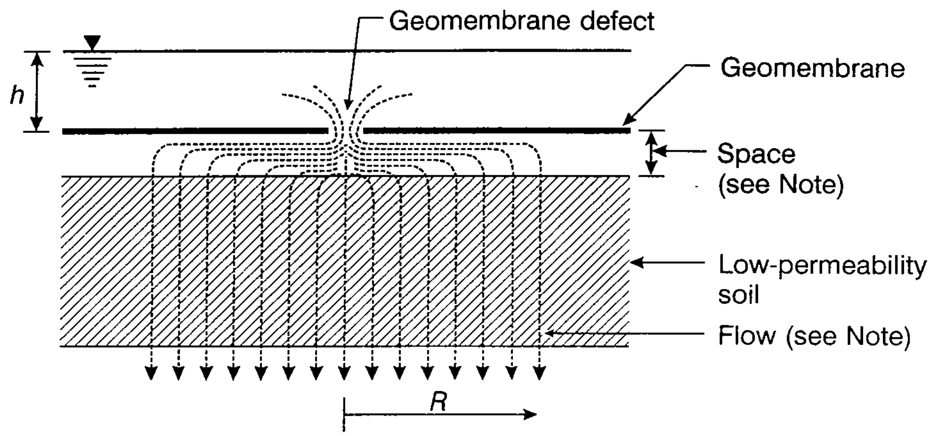

Figure 1. Liquid migration through a composite liner.

Note: The space between the geomembrane and the low-permeability soil is exaggerated to show interface flow. The flow in the soil is assumed to be vertical and R is the radius of the wetted area.

ability soil is called interface flow, and the area covered by the interface flow is called the wetted area.

The quality of the contact between the two components of a composite liner (i.e. the geomembrane and the low-permeability soil) is one of the key factors governing the rate of flow through the composite liner, because it governs the radius of the wetted area (Figure 1). *Good* and *poor* contact conditions have been defined by Bonaparte et al. (1989) as follows:

- *Good* contact conditions correspond to a geomembrane installed, with as few wrinkles as possible, on top of a low-permeability soil layer that has been adequately compacted and has a smooth surface.
- *Poor* contact conditions correspond to a geomembrane that has been installed with a certain number of wrinkles, and/or placed on a low-permeability soil that has not been well compacted and does not appear smooth.

For good contact conditions, it is assumed that there is sufficient compressive stress to maintain the geomembrane in contact with the low-permeability soil layer. In the case of a GCL, good contact conditions may be assumed because GCLs are usually installed flat, and because the bentonite slurry that may exude from a hydrated GCL contributes to establishing a close contact between the geomembrane and the GCL, provided sufficient compressive stress is applied.

Other factors affecting the rate of flow through a composite liner are the size of the defect, the hydraulic conductivity of the low-permeability soil underlying the geomembrane, and the head of liquid on top of the geomembrane. If hydrostatic conditions prevail, the head of liquid on top of the geomembrane is equal to the depth of liquid (Figure 2a) and, if the liquid is unconfined and flowing along a slope (Figure 2b), the head of liquid on top of the geomembrane, h, is given by the following equation:

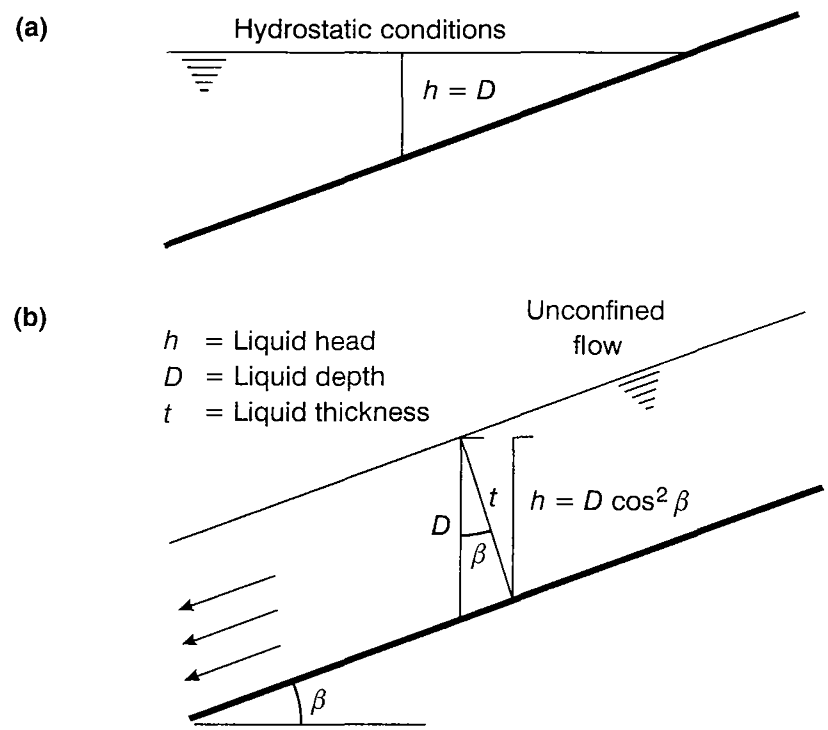

Figure 2. Head of liquid on top of the liner in the case of a liner on a slope: (a) hydrostatic conditions; (b) unconfined flow along the slope.

$$h = t \cos \beta = D \cos^2 \beta \tag{1}$$

where: t = thickness of liquid; D = depth of liquid; and β = slope angle.

1.4 Geomembrane Defects

The following defect shapes are considered in this paper (Figure 3a):

- circular, with a surface area, a, and a diameter, d;
- square, with a side length, b;
- rectangular, with a length, B, and a width, b; and
- infinitely long ($B = \infty$) with a width, b.

1.5 Parameters and Units

The parameters that appear in the liquid migration rate equations are defined in Figure 3b. In the case of a liner on a slope, the liquid head, h, is defined in Figure 2 and by Equation 1.

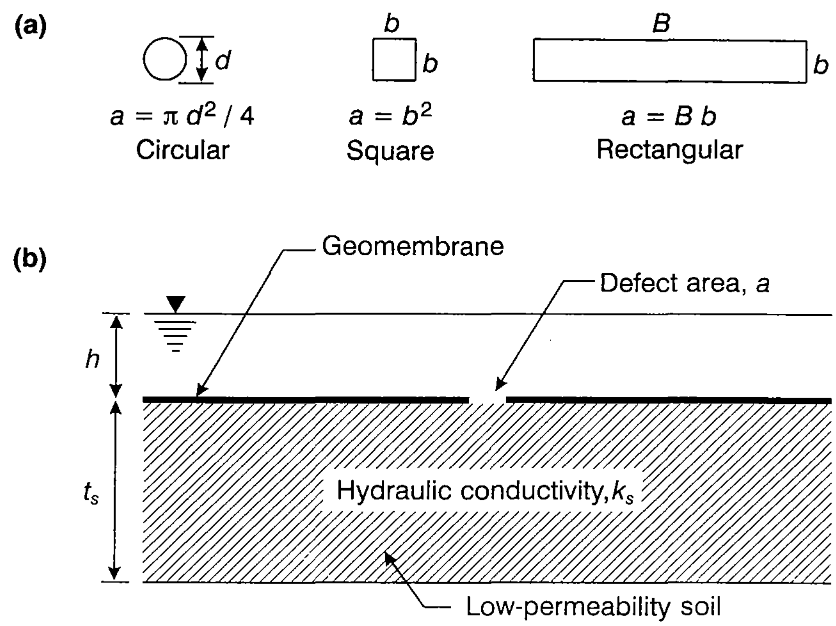

Figure 3. Definition of parameters used in the equations: (a) plan view showing shapes of geomembrane defects; (b) cross section.

Note: If the composite liner is on a slope, the liquid head on top of the liner is defined in Figure 2.

In the equations that follow, Q is the rate of liquid migration through the geomembrane defect. When the defect has an infinite length, the equation gives Q^*, which is the liquid migration rate per unit length of geomembrane defect.

It is important to note that the equations for liquid migration rate that follow are semi-empirical and can only be used with the following basic SI units: h (m), t_s (m), B (m), b (m), a (m^2), k_s (m/s), Q (m^3/s), and Q^* (m^2/s). (Note: t_s = thickness of the low-permeability soil component of the composite liner; k_s = hydraulic conductivity of the low-permeability soil component of the composite liner; and all other symbols were defined above.)

2 EXISTING EQUATIONS TO CALCULATE THE RATE OF LIQUID MIGRATION THROUGH COMPOSITE LINERS

2.1 Equations for Small Head

The following equations have been proposed for the case where the head of liquid on top of the liner is less than the thickness of the low-permeability soil component of the composite liner.

Case of a Circular Defect. In the case of a circular defect, the following equation has been established by Giroud et al. (1989):

$$Q = C_{qo} \ a^{0.1} \ h^{0.9} \ k_s^{0.74} \tag{2}$$

hence:

$$Q = 0.976 \, C_{qo} \ d^{0.2} \, h^{0.9} \, k_s^{0.74} \tag{3}$$

where C_{qo} is the contact quality factor (dimensionless) for a circular or square hole, with:

$$C_{qogood} \leq C_{qo} \leq C_{qopoor} \tag{4}$$

where: C_{qogood} = value of C_{qo} in the case of good contact conditions; and C_{qopoor} = value of C_{qo} in the case of poor contact conditions. (The good and poor contact conditions were defined in Section 1.3.) The following values were established by Giroud et al. (1989):

$$C_{qogood} = 0.21 \tag{5}$$

$$C_{qopoor} = 1.15 \tag{6}$$

Case of a Square Defect. In the case of a square defect, the following equation has been established by Giroud et al. (1992):

$$Q = C_{qo} \ b^{0.2} \ h^{0.9} \ k_s^{0.74} \tag{7}$$

In this case, the value of C_{qo} is the same as in the case of a circular defect discussed above.

Case of a Defect of Infinite Length. In the case of a defect of infinite length ($B = \infty$ in Figure 3a), the following equation has been established by Giroud et al. (1992):

$$Q^* = C_{q\infty} \ b^{0.1} \ h^{0.45} \ k_s^{0.87} \tag{8}$$

where $C_{q\infty}$ is the contact quality factor (dimensionless) for a defect of infinite length, with:

$$C_{q\infty\, good} \leq C_{q\infty} \leq C_{q\infty\, poor} \tag{9}$$

where: $C_{q\infty good}$ = value of $C_{q\infty}$ in the case of good contact conditions; and $C_{q\infty poor}$ = value of $C_{q\infty}$ in the case of poor contact conditions. The following values were established by Giroud et al. (1992):

$$C_{q\infty\, good} = 0.52 \tag{10}$$

$$C_{q\infty\, poor} = 1.22 \tag{11}$$

Case of a Rectangular Defect. In the case of a rectangular defect, the following equation has been established by Giroud et al. (1992):

$$Q = C_{qo}\ b^{0.2}\ h^{0.9}\ k_s^{0.74} + C_{q\infty}(B-b)\ b^{0.1}\ h^{0.45}\ k_s^{0.87} \tag{12}$$

where C_{qo} and $C_{q\infty}$ have the values defined above.

2.2 Equations for Large Head

When the head of liquid on top of the liner is greater than the thickness of the low-permeability soil component of the composite liner, Equations 2, 3, 7, 8 and 12 are not valid. Instead, the following equations should be used, as shown by Giroud et al. (1992):

- Circular defect:

$$Q = C_{qo}\ i_{avgo}\ a^{0.1}\ h^{0.9}\ k_s^{0.74} \tag{13}$$

- Square defect:

$$Q = C_{qo}\ i_{avgo}\ a^{0.2}\ h^{0.9}\ k_s^{0.74} \tag{14}$$

- Infinitely long defect:

$$Q^* = C_{q\infty}\ i_{avg\infty}\ b^{0.1}\ h^{0.45}\ k_s^{0.87} \tag{15}$$

- Rectangular defect:

$$Q = C_{qo}\ i_{avgo}\ b^{0.2}\ h^{0.9}\ k_s^{0.74} + C_{q\infty}\ i_{avg\infty}\ (B-b)\ b^{0.1}\ h^{0.45}\ k_s^{0.87} \tag{16}$$

where: i_{avgo} = average hydraulic gradient in the low-permeability soil in the case of a circular or square defect; and $i_{avg\infty}$ = average hydraulic gradient in the low-permeability soil in the case of a defect of infinite length. The values of i_{avgo} and $i_{avg\infty}$ are given in the graphs presented in Figure 4.

It appears that, when the head of liquid is greater than the thickness of the low-permeability soil component of the composite liner, the calculation of the rate of liquid migra-

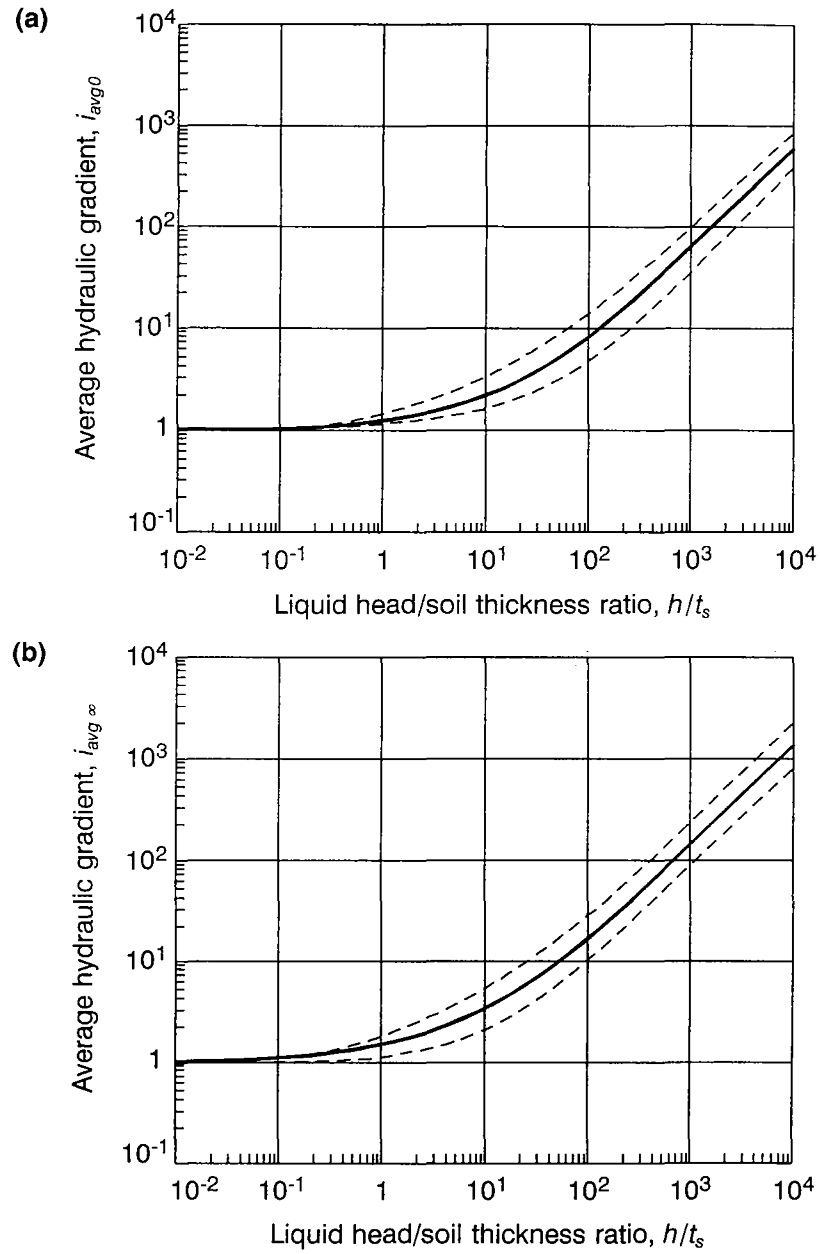

Figure 4. Values of the average hydraulic gradient to be used in equations for liquid migration rate calculations: (a) case of a circular or square defect; (b) case of an infinitely long defect (from Giroud et al. 1992).

Note: The dashed lines represent the upper and lower limit of the range of values for i_{avgo} and $i_{avg\infty}$, since for a given value of h/t_s there is not a unique value of i_{avgo} and $i_{avg\infty}$ (Giroud et al. 1992). The solid lines represent the curves given by Equation 17 (Figure 4a) and Equation 18 (Figure 4b).

tion is not entirely analytical since graphs must be used. This is not convenient, especially when calculations for a large number of cases have to be performed.

3 NEW EQUATIONS TO CALCULATE THE RATE OF LIQUID MIGRATION THROUGH COMPOSITE LINERS

3.1 Analytical Expression of the Average Hydraulic Gradient

After numerous attempts, it was found that a good approximation of the values of i_{avgo} and $i_{avg\infty}$ presented in Figure 4 is given by the following equations:

$$i_{avgo} = 1 + 0.1 (h / t_s)^{0.95} \tag{17}$$

$$i_{avg\infty} = 1 + 0.2 (h / t_s)^{0.95} \tag{18}$$

3.2 New Equations for Liquid Migration Rate

Combining Equations 13 to 16 with Equations 17 and 18 gives the following equations that can be used to calculate the rate of liquid migration through composite liners:

- Circular defect:

$$Q = C_{qo} \; [1 + 0.1 \, (h / t_s)^{0.95}] \, a^{0.1} \; h^{0.9} \; k_s^{0.74} \tag{19}$$

hence:

$$Q = 0.976 \; C_{qo} \left[1 + 0.1 (h / t_s)^{0.95}\right] d^{0.2} \, h^{0.9} \, k_s^{0.74} \tag{20}$$

- Square defect:

$$Q = C_{qo} \; [1 + 0.1 \, (h / t_s)^{0.95}] \, b^{0.2} \; h^{0.9} \; k_s^{0.74} \tag{21}$$

- Infinitely long defect:

$$Q^* = C_{q\infty} \; [1 + 0.2 \, (h / t_s)^{0.95}] \, b^{0.1} \; h^{0.45} \; k_s^{0.87} \tag{22}$$

- Rectangular defect:

$$Q = C_{qo} \; [1 + 0.1 \, (h / t_s)^{0.95}] \, b^{0.2} \; h^{0.9} \; k_s^{0.74} + C_{q\infty} [1 + 0.2 \, (h / t_s)^{0.95}] \, (B - b) \, b^{0.1} \; h^{0.45} \; k_s^{0.87} \tag{23}$$

Values of C_{qo} are given by Equations 4, 5 and 6. Values of $C_{q\infty}$ are given by Equations 9, 10 and 11. The other parameters are defined in Section 1.5. Equations 19 to 23 are semi-empirical and they must be used with the units defined in Section 1.5.

It should be noted that, when the head of liquid on top of the liner is smaller than the thickness of the low-permeability soil component of the composite liner, the value of i_{avgo} and $i_{avg\infty}$ given by Equations 17 and 18, respectively, is approximately equal to 1, and Equations 19, 20, 21, 22 and 23 become identical to Equations 2, 3, 7, 8 and 12, respectively.

3.3 Limitations

The limits of validity of the above equations are discussed in detail by Giroud et al. (1997). These limits can be summarized as follows:

- If the defect is circular, the defect diameter should be no less than 0.5 mm and not greater than 25 mm. In the case of defects that are not circular, it is proposed to use these limitations for the defect width.
- The liquid head on top of the geomembrane should be equal to or less than 3 m.
- The hydraulic conductivity of the low-permeability soil underlying the geomembrane should be equal to or less than a certain value k_G , which is less than the value k_{GB} for which the relevant equation for the considered defect type (i.e. an equation selected from Equations 19 to 23) and Bernoulli's equation for free flow through an orifice give the same value of the rate of liquid migration through the geomembrane defect.

To ensure a smooth transition between liquid migration rates calculated using Equations 19 to 23 and those calculated using Bernoulli's equation, Giroud et al. (1997) propose the following value for k_G :

$$k_G = k_{GB}/10 \tag{24}$$

In the case where the geomembrane defect is circular, k_G is given by the following equation (Giroud et al. 1997):

$$k_G = \left\{ \frac{0.3891\, d^{1.8}}{C_{qo} \left[1 + 0.1 \left(\frac{h}{t_s} \right)^{0.95} \right] h^{0.4}} \right\}^{1/0.74} \tag{25}$$

Equation 25 must be used with the units defined in Section 1.5. Values of k_G calculated using Equation 25 with $C_{qo} = 0.21$ (i.e. good contact conditions, as indicated by Equation 5) and $t_s = 0.6$ m are given in Table 1.

Table 1. Hydraulic conductivity, k_G, of the low-permeability soil underlying the geomembrane that gives the upper limit of validity of the equation for liquid migration through a circular defect in a geomembrane underlain by a low-permeability soil (Equation 20).

Head of liquid on top of the geomembrane, h (m)	Geomembrane defect diameter, d (mm)						
	0.5	1	2	3	5	10	11.284
0.01	2.6×10^{-7}	1.4×10^{-6}	7.5×10^{-6}	2.0×10^{-5}	7.0×10^{-5}	3.8×10^{-4}	5.1×10^{-4}
0.03	1.4×10^{-7}	7.7×10^{-7}	4.1×10^{-6}	1.1×10^{-5}	3.8×10^{-5}	2.1×10^{-4}	2.8×10^{-4}
0.1	7.3×10^{-8}	3.9×10^{-7}	2.1×10^{-6}	5.7×10^{-6}	2.0×10^{-5}	1.1×10^{-4}	1.4×10^{-4}
0.3	3.8×10^{-8}	2.1×10^{-7}	1.1×10^{-6}	3.0×10^{-6}	1.0×10^{-5}	5.6×10^{-5}	7.5×10^{-5}
1	1.8×10^{-8}	9.5×10^{-8}	5.1×10^{-7}	1.4×10^{-6}	4.7×10^{-6}	2.6×10^{-5}	3.4×10^{-5}
3	7.1×10^{-9}	3.8×10^{-8}	2.1×10^{-7}	5.6×10^{-7}	1.9×10^{-6}	1.0×10^{-5}	1.4×10^{-5}

Notes: The tabulated values of k_G were calculated using Equation 25 with $C_{qo} = 0.21$ (good contact) and t_s = 0.6 m. The defect diameter of 11.284 mm corresponds to a defect surface area of 1 cm^2.

3.4 Example

A composite liner consists of a geomembrane placed on a GCL having a thickness of 6 mm and a hydraulic conductivity of 2×10^{-11} m/s. The geomembrane has a rectangular defect with a width of 1 mm and a length of 15 mm. The head of liquid on top of the composite liner is 25 mm. Calculate the rate of liquid migration through this defect.

The rate of liquid migration through the composite liner is calculated as follows using Equation 23:

$$Q = C_{qo}[1 + 0.1(25/6)^{0.95}](1\times10^{-3})^{0.2}(25\times10^{-3})^{0.9}(2\times10^{-11})^{0.74} + C_{q\infty}[1 + 0.2(25/6)^{0.95}](15-1)\times10^{-3}(1\times10^{-3})^{0.1}(25\times10^{-3})^{0.45}(2\times10^{-11})^{0.87}$$

hence:

$$Q(\mathrm{m}^3/\mathrm{s}) = C_{qo}(1.53\times10^{-10}) + C_{q\infty}(1.17\times10^{-12})$$

Assuming good contact between the geomembrane and the GCL, Equations 5 and 10 give:

$$C_{qo} = 0.21 \qquad C_{q\infty} = 0.52$$

hence:

$$Q(\text{m/s}^3) = 0.21 \times 1.53 \times 10^{-10} + 0.52 \times 1.17 \times 10^{-12} = 3.21 \times 10^{-11} + 6.08 \times 10^{-13}$$

hence:

$$Q = 3.27 \times 10^{-11} \text{ m/s}^3 = 2.8 \times 10^{-3} \text{ liters/day} = 1.0 \text{ liter/year}$$

It is interesting to note that, if the defect had been square with a side length of 1 mm, the rate of liquid migration through the defect would have been expressed by the first term of the above equation (3.21×10^{-11} m^3/s), which is much greater than the second term. In other words, the calculated rate of liquid migration is only slightly greater through the 15 mm × 1 mm defect than through the 1 mm × 1 mm defect. This is because, in this particular example, the radius of the wetted area, calculated as indicated by Giroud et al. (1992), is very large compared to the defect size and is far more dependent on defect width than on defect length. (The calculation gives a wetted area radius of approximately 0.6 m.)

4 CONCLUSIONS

The equations presented in this paper provide design engineers with an entirely analytical method to calculate the rate of liquid migration through a composite liner, due to geomembrane defects, for liquid heads on top of the liner up to 3 m. The new equations are equivalent to the existing method (Giroud et al. 1992, 1994) which requires both equations and graphs when the head of liquid on top of the liner is greater than the thickness of the low-permeability soil component of the composite liner. However, the new equations are more convenient because the values that had to be obtained from graphs are now incorporated into the equations (Equations 19 to 23).

The new equations are particularly useful in cases where the low-permeability soil component of the composite liner is a GCL because the head of liquid on top of the liner is often greater than the thickness of the GCL.

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NOTATIONS

Basic SI units are given in parentheses.

a	=	defect area (m^2)
B	=	length of rectangular defect (m)
b	=	width of rectangular defect (m)
b	=	side length of square defect (m)
C_q	=	contact quality factor (dimensionless)
C_{qo}	=	contact quality factor for a circular or square defect (dimensionless)
C_{qogood}	=	value of C_{qo} in the case of good contact conditions (dimensionless)
C_{qopoor}	=	value of C_{qo} in the case of poor contact conditions (dimensionless)
$C_{q\infty}$	=	contact quality factor for a defect of infinite length (dimensionless)
$C_{q\infty\,good}$	=	value of $C_{q\infty}$ in the case of good contact conditions (dimensionless)
$C_{q\infty\,poor}$	=	value of $C_{q\infty}$ in the case of poor contact conditions (dimensionless)
D		depth of liquid on top of the geomembrane (m)
d	=	diameter of circular defect (m)
h	=	head of liquid on top of the geomembrane (m)
i_{avgo}	=	average hydraulic gradient in the low-permeability soil in the case of a circular or square defect (dimensionless)
$i_{avg\infty}$	=	average hydraulic gradient in the low-permeability soil in the case of an infinitely long defect (dimensionless)
k_G	=	value of k_s above which Equations 19 to 23 are not valid (m/s)
k_{GB}	=	value of k_s for which Equation 19 to 23 and Bernoulli's equation for free flow through an orifice give the same value of the rate of liquid migration through a geomembrane defect (m/s)

k_s	=	hydraulic conductivity of the low-permeability soil component of the composite liner (m/s)
Q	=	liquid migration rate through the considered geomembrane defect (m^3/s)
Q^*	=	liquid migration rate per unit length of geomembrane defect in the case of an infinitely long defect (m^2/s)
R	=	radius of wetted area (m)
t	=	thickness of liquid on top of the geomembrane (m)
t_s	=	thickness of the low-permeability soil component of the composite liner (m)
β	=	slope angle (°)